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Application of FEM to Estimate Complex Permittivity of Dielectric Material at Microwave Frequency Using Waveguide Measurements

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Symbols

a	x-dimension of rectangular waveguide
a_0	dominant mode reflection coefficient
a_0'	measured reflection coefficient
a_p	complex waveguide modal amplitude of p^{th} mode
b	y-dimension of rectangular waveguide
b_m	complex amplitude associated with tetrahedra basis function
[b]	column matrix
$\vec{E}\big\vert_{over\ S_1}$	tangential electric field vector over the plane P_1
$\vec{E}^I(x, y, z)$	transverse electric field vector in Region I
$\vec{E}^{II}(x, y, z)$	electric field vector in Region II
$\vec{e}_p$	rectangular waveguide vector modal function for pth mode
$f_1(\varepsilon_r', \varepsilon_r'')$	real function of $\varepsilon_r', \varepsilon_r''$
$f_2(\varepsilon_r', \varepsilon_r'')$	real function of $\varepsilon_r', \varepsilon_r''$
$\frac{\partial f_1}{\partial \varepsilon_1'}$	partial derivative of f_1 with respect to ε_1'
$\frac{\partial f_1}{\partial \varepsilon_1''}$	partial derivative of f_1 with respect to ε_1''
$\frac{\partial f_2}{\partial \varepsilon_1'}$	partial derivative of f_2 with respect to ε_1'
$\frac{\partial f_2}{\partial \varepsilon_1''}$	partial derivative of f_2 with respect to ε_1''
$\vec{H}^I x, y, z)$	transverse magnetic field vector in Region I
$\vec{H}^{II}(x, y, z)$	magnetic field vector in Region II
$\vec{h}_p$	rectangular waveguide vector modal function for p^{th} mode
j	$\sqrt{-1}$

k_0	free-space wave number
$[S]$	global finite element matrix
$[S_{el}]$	element matrix for single tetrahedral element
S_1	surface area over plane P_1
$\vec{T}(x, y, z)$	vector testing function
TE_{10}	dominant mode
$[v]$	excitation column vector
w_x, w_y, w_z	x-,y-, and z-dimensions of rectangular sample
x, y, z	Cartesian Coordinate system
Y_p^I	modal admittance of p^{th} mode
ε_0	permittivity of free-space
μ_0	permeability of free-space
ε_r	relative permittivity of medium in Region II
μ_r	relative permeability of medium in Region II
ε_r'	real part of ε_r
ε_r''	imaginary part of ε_r
$d\varepsilon_r'$	small increment in ε_r'
$d\varepsilon_r''$	small increment in ε_r''
μ_r'	real part of μ_r
μ_r''	imaginary part of μ_r
Δ_1 and Δ_2	small quantities
γ_p^I	propagation constant for p^{th} mode
ω	angular frequency

Abstract

A simple waveguide measurement technique is presented to determine the complex dielectric constant of a dielectric material. The dielectric sample is loaded in a shorted x-band rectangular waveguide. Using a network analyzer, the reflection coefficient of the shorted waveguide (loaded with sample) is measured. Using the Finite Element Method (FEM) the exact reflection coefficient of the shorted waveguide (loaded with the sample) is determined as a function of the dielectric constant. Matching the measured value of the reflection coefficient with the reflection coefficient calculated using the FEM utilizing the Newton-Raphson Method, an estimate of the dielectric constant of a dielectric material is obtained. A comparison of estimated values of dielectric constant obtained from simple waveguide modal theory and the present approach is presented.

I Introduction

Application of materials in aerospace, microwave, microelectronics and communication industries requires the exact knowledge of material parameters such as permittivity and permeability. Over the years many methods have been developed and used for measuring permittivity (ε_r', ε_r'') and permeability (μ_r', μ_r'') of materials [1]. The most accurate measurement at high frequency can be done using the high Q resonant cavity technique [2]. However, the main disadvantage of the cavity method is that the measured results are applicable only over a narrow frequency band [2]. Electric properties of material over wide range of frequencies can be done with less accuracy using the transmission line methods. In the transmission line method an isotropic material sample with specific length is positioned in a transmission line and the $(\varepsilon_r', \varepsilon_r'')$ and (μ_r', μ_r'') are determined from the measured reflection and transmission coefficients. The material sample used in these measurements is usually of a cross section which is the same as that of the trasmission line. The uniform cross section of the sample is selected so that a dominant mode analysis is sufficient and accurate for measuring the material constants. However, when the sample selected is not of uniform cross section or the sample occupies only a part of trasmission line cross section, then the complete modal analysis is required to measure accurately the material properties. The complete modal analysis is quite complicated, if not impossible. In such cases, when the sample cross section is different from that of the transmission line, a numerical method such as the FEM instead of the modal analysis is much easier to implement to obtain material properties.

In the present report, the FEM is proposed to estimate complex permittivity of material using a terminated rectangular waveguide. The method described here may be easily extended to estimate the complex permeability of material. The material sample of specific length but of arbitrary cross section is assumed to be present in a shorted rectangular waveguide. The reflection coefficient at some arbitrary selected reference plane in the rectangular waveguide is measured at a given freqeuency. Since only determination of permittivity is required, a single reflection coefficent measurement suffices. The reflection coefficient at the given frequency is also calculated as a function of $(\varepsilon_r', \varepsilon_r'')$ using the FEM. From the calculated and measured values of reflection coefficients and the use of Newton-Raphson Method [3], the complex permittivity of the given material is then determined. The complex permittivities of Teflon and Plexiglass obtained using the present techniques are compared with the values obtained using the standard software available with the *hp (Hewlett Packard)*-8510 Network Analyze [6-7].

II Theory

(a) Direct Problem

In this section the FEM is used to determine the reflection coefficient of a short circuited rectangular transmission line loaded with an arbitrary shaped dielectric material. Figure 1 shows a terminated rectangular waveguide with a dielectric sample of arbitrary cross section. It is assumed that the waveguide is excited by a dominant TE_{10} mode from the right and the reflection coefficient is measured at the reference plane P_1 as shown in figure 1(a). For the purpose of analysis the problem is divided into two regions: Region I ($z < 0$) and Region II ($z > 0$). Using the waveguide vector modal functions, the transverse electromagnetic fields in the region I are expressed as [4]

$$\vec{E}^{I}(x,y,z) = \vec{e}_0(x,y)\, e^{-j\gamma_0^I z} + \sum_{p=0}^{\infty} a_p \cdot \vec{e}_p(x,y)\, e^{j\gamma_p^I z} \tag{1}$$

$$\vec{H}^{I}(x,y,z) = \vec{h}_0(x,y)\, Y_0^I\, e^{-j\gamma_0^I z} - \sum_{p=0} a_p \cdot \vec{h}_p(x,y)\, Y_p^I\, e^{j\gamma_p^I z} \tag{2}$$

In deriving equation (1) and (2) it is assumed that only the dominant mode is incident on the interface P_1 and a_p is the amplitude of reflected modes at z=0 plane. $\vec{e}_0(x,y)$ and $\vec{h}_0(x,y)$ are the rectangular waveguide vector modal functions for the dominant mode and $\vec{e}_p(x,y)$ $\vec{h}_p(x,y)$ are the rectangular waveguide vector modal function for p^{th} mode [4]. Y_p^I and γ_p^I appearing in equations (1) and (2) are the characteristic admittance and propagation constant for p^{th} mode and are given by

$$\mathrm{Y}_p^I = \frac{j\ \omega\ \varepsilon_0}{\gamma_p^I}$$

$$\gamma_p^I = \sqrt{k_0^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \quad \text{for } \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \le k_0^2$$

$$= -j\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k_0^2} \text{ for } \left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right) > k_0^2$$

where m, n are the mode indices, a, b are the x- and y-dimensions, respectively, of the rectangular waveguide, and k_0 is the free-space wave number. The unknown complex modal amplitude a_p may be obtained in terms of the transverse electric field over the plane P_1 as follows

$$1 + a_0 = \iint_{S_1} \vec{\mathrm{E}}\Big|_{over\ \mathrm{S}_1} \bullet \vec{e}_0 ds \tag{3}$$

$$a_p = \iint_{S_1} \vec{\mathrm{E}}\Big|_{over\ \mathrm{S}_1} \bullet \vec{e}_p ds \tag{4}$$

where $\vec{\mathrm{E}}\Big|_{over\ \mathrm{S}_1}$ is a tangential electric field over the surface S_1 at the reference plane P_1.

The electromagnetic field inside Region II is obtained using the FEM formulation [5]. The vector wave equation for the $\vec{E}^{II}$ field is given by

$$\nabla\times\left(\frac{1}{\mu_r}\cdot\nabla\times\vec{E}^{II}\right) - \left(k_0^2\varepsilon_r\right)\vec{E}^{II} = 0 \tag{5}$$

Multiplying equation (5) by a testing function $\vec{T}(x, y, z) = \hat{x}T_x + \hat{y}T_y + \hat{z}T_z$ and integrating over the volume of the Region II, equation (5) yields with simple vector manipulations

$$\iiint_V \left[\nabla\times\vec{T} \bullet \left(\frac{1}{\mu_r} \nabla\times\vec{E}^{II} \right) - \left(k_0^2 \varepsilon_r \right) \vec{E}^{II} \bullet \vec{T} \right] dv$$

$$= \quad -\iint_{S_1} \vec{T} \bullet \left(\hat{n} \times \frac{1}{\mu_r} \nabla\times\vec{E}^{II} \right) ds \qquad (6)$$

where V is the volume of Region II and $\hat{n}$ is the unit normal vector to the surface S_1 drawn outward with respect to the volume V. Using $\nabla\times\vec{E}^{II} = -j\omega\mu_0\vec{H}^{II}$, equation (6) may be rewritten as

$$\iiint_V \left(\nabla\times\vec{T} \bullet \left(\frac{1}{\mu_r} \cdot \nabla\times\vec{E}^{II} \right) - \left(k_0^2 \varepsilon_r \right) \vec{E}^{II} \bullet \vec{T} \right) dv$$

$$= \quad -\iint_{S_1} \vec{T} \bullet \left(\hat{n} \times \left(\frac{-j\omega\mu_0}{\mu_r} \right) \vec{H}^{II} \right) ds \qquad (7)$$

Since $\left(\vec{H}^{I} \Big|_{over\ S_1} \right)_{\tan} = \left(\vec{H}^{II} \Big|_{over\ S_1} \right)_{\tan}$ and $\vec{\mathrm{E}} \Big|_{over\ S_1} = \left(\vec{E}^{II} \Big|_{over\ S_1} \right)_{\tan}$, the integral on right hand side of equation (7) may be written as

$$-\iint_{S_1} \vec{T} \bullet \left(\hat{n} \times \left(\frac{-j\omega\mu}{\mu_r} \right) \vec{H}^{II} \right) ds = 2 \left(\frac{j\omega\mu_0}{\mu_r} \right) \cdot \ \mathrm{Y}_0^I \cdot \iint_{S_1} \vec{T} \bullet \vec{e}_0\,(x, y)\, ds$$

$$-\left(\frac{j\omega\mu}{\mu_r} \right) \cdot \left(\sum_{p=0}^{\infty} \mathrm{Y}_p^I \left(\iint_{S_1} \vec{T} \bullet \vec{e}_p\,(x, y)\, ds \cdot \iint_{S_1} \vec{E}^{II} \Big|_{over\ S_1} \bullet \vec{e}_p\,(x, y)\, ds \right) \right) \qquad (8)$$

Substituting (8) in (7), we obtain

$$\iiint_V \left(\nabla\times\vec{T} \bullet \left(\frac{1}{\mu_r} \cdot \nabla\times\vec{E}^{II} \right) - \left(k_0^2 \varepsilon_r \right) \vec{E}^{II} \bullet \vec{T} \right) dv = 2 \left(\frac{j\omega\mu}{\mu_r} \right) \cdot \ \mathrm{Y}_0^I \cdot \iint_{S_1} \vec{T} \bullet \vec{e}_0\,(x, y)\, ds$$

$$-\left(\frac{j\omega\mu}{\mu_r} \right) \cdot \left(\sum_{p=0}^{\infty} \mathrm{Y}_p^I \left(\iint_{S_1} \vec{T} \bullet \vec{e}_p\,(x, y)\, ds \cdot \iint_{S_1} \vec{E}^{II} \Big|_{over\ S_1} \bullet \vec{e}_p\,(x, y)\, ds \right) \right) \qquad (9)$$

In order to solve the equation (9), the volume enclosed by Region II is discretized by using first-order tetrahedral elements such as the one shown in figure 2. The electric field in a single tetrahedral element is represented as

$$\vec{E}^{II} = \sum_{m=1}^{6} b_m \cdot \vec{W}_m \tag{10}$$

where b_m , m=1,2,3..6 are the six complex amplitudes of electric field associated with the six edges of the tetrahedron, and $\vec{W}_m(x, y, z)$ is the vector basis function associated with the mth edge of the tetrahedron. Detail derivation for the expression for $\vec{W}_m(x, y, z)$ is given in reference [5]. Substituting equation (10) into equation (9), integration over the volume of one tetrahedron results in

$$\frac{1}{\mu_r} \cdot \sum_{m=1}^{6} b_m \cdot \iiint_V \left(\nabla\times\overrightarrow{W_n} \bullet \nabla\times\overrightarrow{W_m} - k_0^2 \varepsilon_r \vec{W}_m \bullet \vec{W}_n \right) dv = 2\left(\frac{j\omega\mu}{\mu_r}\right) \cdot \mathrm{Y}_0^I \cdot \iint_{S_1} \overrightarrow{W_n} \bullet \vec{e}_0(x, y)\, ds$$

$$-\left(\frac{j\omega\mu}{\mu_r}\right) \cdot \sum_{m=1}^{6} b_m \cdot \sum_{p=0}^{\infty} \mathrm{Y}_p^I \left(\iint_{S_1} \vec{W}_n \bullet \vec{e}_p(x, y)\, ds \cdot \iint_{S_1} \vec{W}_m \bullet \vec{e}_p(x, y)\, ds \right) \tag{11}$$

The above equation can be written in a matrix form

$$[S_{el}] \cdot [b] = [v] \tag{12}$$

where the elements of element matrices are given by

$$S_{el}(m, n) = \frac{1}{\mu_r} \cdot \iiint_V \left(\nabla\times\overrightarrow{W_n} \bullet \nabla\times\overrightarrow{W_m} - k_0^2 \varepsilon_r \vec{W}_m \bullet \vec{W}_n \right) dv$$

$$+ \left(\frac{j\omega\mu}{\mu_r}\right) \cdot \sum_{p=0}^{\infty} \mathrm{Y}_p^I \left(\iint_{S_1} \vec{W}_n \bullet \vec{e}_p(x, y)\, ds \cdot \iint_{S_1} \vec{W}_m \bullet \vec{e}_p(x, y)\, ds \right) \tag{13}$$

$$v(n) = 2\left(\frac{j\omega\mu}{\mu_r}\right) \cdot \mathrm{Y}_0^I \cdot \iint_{S_1} \overrightarrow{W_n} \bullet \vec{e}_0(x, y)\, ds \tag{14}$$

These element matrices can be assembled over all the tetrahedron elements in Region II to obtain a global matrix equation

$$[S] \cdot [b] = [v] \tag{15}$$

The solution vector $[b]$ of the matrix equation (15) is then used to determine reflection coefficient at the reference plane P_1 as

$$a_0 = \iint_{S_1} \vec{E}\Big|_{over\ S_1} \bullet \vec{e}_0 ds - 1 \tag{16}$$

(b) Rectangular Waveguide Measurement System

The reflection coefficient a_0' due to a terminated reactangular waveguide loaded with a given material sample piece can be measured using the procedure described in [6]. Assuming the sample piece occupies the entire cross section of the waveguide, an algorithm which uses the Nicholson-Ross Technique [6-7] is used to determine the complex permittivity of the sample. However, the algorithm which is based on the Nicholson-Ross Technique cannot be used when the sample piece occupies part of the cross section of rectangular waveguide. Also the Nicholson-Ross Technique can only be used for a through measurement and not for a measurement with terminated waveguide. When the dielectric sample is of arbitrary shape the procedure described in the next section is used.

(c) Inverse Problem

This section presents computations of complex dielectric constant of a given sample piece from a one port measurement of the reflection coefficient. From the given geometry of the sample and its position in the terminated rectangular waveguide, the reflection coefficient

$a_0(\varepsilon_r', \varepsilon_r'')$ is calculated using the FEM for assumed values of $(\varepsilon_r', \varepsilon_r'')$. If a_0' is the measured reflection coefficient then the error in the calculated value of reflection coefficient is $a_0(\varepsilon_r', \varepsilon_r'') - a_0'$. Writing the error in real and imaginary part we get

$$f_1(\varepsilon_r', \varepsilon_r'') = \text{real}\,(a_0(\varepsilon_r', \varepsilon_r'') - a_0') \tag{17}$$

$$f_2(\varepsilon_r', \varepsilon_r'') = \text{imag}\,(a_0(\varepsilon_r', \varepsilon_r'') - a_0') \tag{18}$$

If $(\varepsilon_r', \varepsilon_r'')$ are incremented by small values to $(\varepsilon_r' + d\varepsilon_r', \varepsilon''_r + d\varepsilon''_r)$ such that $f_1(\varepsilon_r' + d\varepsilon_r', \varepsilon''_r + d\varepsilon''_r)$ and $f_2(\varepsilon_r' + d\varepsilon_r', \varepsilon''_r + d\varepsilon''_r)$ are simultaneously zero then we can write following matrix equation [3]

$$\begin{bmatrix} \dfrac{\partial f_1}{\partial (d\varepsilon_r')} & \dfrac{\partial f_1}{\partial (d\varepsilon''_r)} \\ \dfrac{\partial f_2}{\partial (d\varepsilon_r')} & \dfrac{\partial f_2}{\partial (d\varepsilon''_r)} \end{bmatrix} \cdot \begin{bmatrix} d\varepsilon_r' \\ d\varepsilon''_r \end{bmatrix} = \begin{bmatrix} f_1(\varepsilon_r', \varepsilon_r'') \\ f_2(\varepsilon_r', \varepsilon_r'') \end{bmatrix} \tag{19}$$

From the solution of equation (18), new modified values of $(\varepsilon_r', \varepsilon_r'')$ are obtained as

$$(\varepsilon_r')_{new} = \varepsilon_r' + d\varepsilon_r' \tag{20}$$

$$(\varepsilon_r'')_{new} = \varepsilon_r'' + d\varepsilon_r'' \tag{21}$$

The reflection coefficient $a_0(\varepsilon_r', \varepsilon_r'')$ with modified values of $(\varepsilon_r', \varepsilon_r'')$ is again calculated using the FEM procedure. With the new value of $a_0(\varepsilon_r', \varepsilon_r'')$ computation through equations (17)- (19) are repeated. The above procedure is repeated until required convergence is obtained (i.e. $d\varepsilon_r' \le \Delta_1$ and $d\varepsilon_r'' \le \Delta_2$, where Δ_1 and Δ_2 are preselected small quantities). The procedure described above will converge to the true value of complex permittivity if the first choice of $(\varepsilon_r', \varepsilon_r'')$ is close to the true values of complex permittivity.

III Numerical Results

For numerical result , samples made from Teflon and Plexiglass materials are considered. First, samples which occupy the entire volume enclosed by Region II are considered. This is done for the purpose of comparison with results obtained by the present technique and the results obtained by standard software available with the *hp(*-8510 Network Analyzer. Numerical results obtained from samples which do not occupy entire volume of the Region II are also presented.

First, a sample piece of rectangular shape with dimension w_x = 2.29 cm, w_y = 1.0 cm, w_z = 0.947 cm was cut from a Teflon sheet. This size was selected so that the sample piece occupies entire volume of the Region II. The reflection coefficient at reference plane P_1 was measured by placing the sample piece in a x-band rectangular waveguide and using *hp*-8510 network analyzer over the frequency band 8.2GHz-12.40GHz. Using the procedure described in section II, the complex permittivity of the Teflon sample is calculated and shown in figure 3. For the same sample complex permittivity using the Nicholson-Ross Technique is also calculated and presented in figure 3. The two results agree well except at a frequency close to 12 GHz where small resonance is detected by the FEM procedure. Because of very low loss characteristic of the Teflon material, the estimation of imaginary part of complex permittivity is not very reliable. The complex permittivity of Plexiglass is also obatined using the present procedure and presented in figure 4 along with the results obtained by Nicholson-Ross Technique [7]. The two results are in good agreement with each other.

In order to validate the present method for a material sample for an arbitrary shaped sample, a Teflon sample with dimension $w_x = 1.58$ cm, $w_y = 0.785$cm, $w_z = 0.632$cm is cut from a Teflon sheet. The sample is then placed in a rectangular waveguide for measurement of reflection coefficient as shown in figure 5. From the measured value of reflection coefficient and following the procedure described in section II the complex permittivity is calculated and presented in figure 6. For comparison, the complex permittivity obtained using full size sample (i.e. $w_x = 2.29$ cm, $w_y = 1.0$ cm, $w_z = 0.947$ cm) is also presented in figure 6. From figure 6 it may be concluded that the FEM procedure can be used to determine the complex permittivity of dielectric material using an arbitrarily shaped sample.

Figure 7 shows the real part of complex permittivity of Plexiglass obtained using a sample with dimension $w_x = 1.58$ cm, $w_y = 0.785$cm, $w_z = 0.632$cm and placed in Region II as shown in figure 5. The real part of complex permittivity of Plexiglass obtained using full size sample is also presented in figure 7. The two results are in good agreement within a range of uncertainty specified by manufacturer.

IV Conclusion

A FEM procedure in conjunction with the Newton-Raphson Method has been presented to determine complex permittivity of a dielectric material using an arbitrarily shaped sample. The arbitrarily shaped sample of a given dielectric material is placed in a terminated x-band rectangular waveguide. The reflection coefficent at a reference plane is measured with *hp*-8510 Network Analyzer. For the same configuration of a terminated x-band waveguide loaded with the sample piece, the reflection coefficient is calculated as a function of complex dielectric constant using the FEM . The Newton-Raphson Method is then used to determine the complex dielectric constant by matching the calculated value with the measured value. The measured values of complex permittivity of Teflon and Plexiglass using the FEM method are in good agreement with the results obtained by the Nicholson-Ross Technique.

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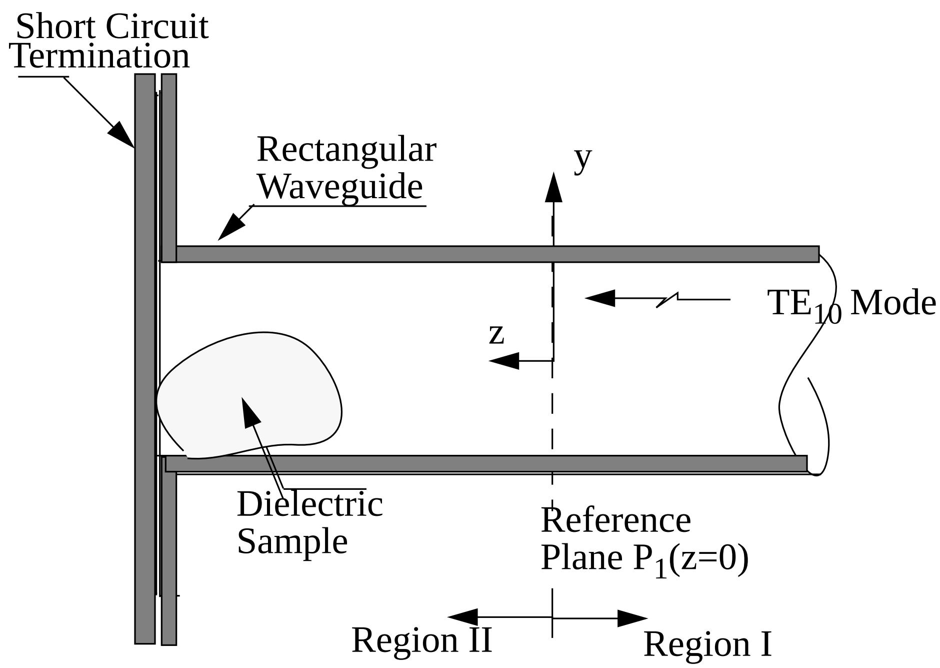

Figure 1(a) Longitudinal view of rectangular waveguide loaded with dielectric sample

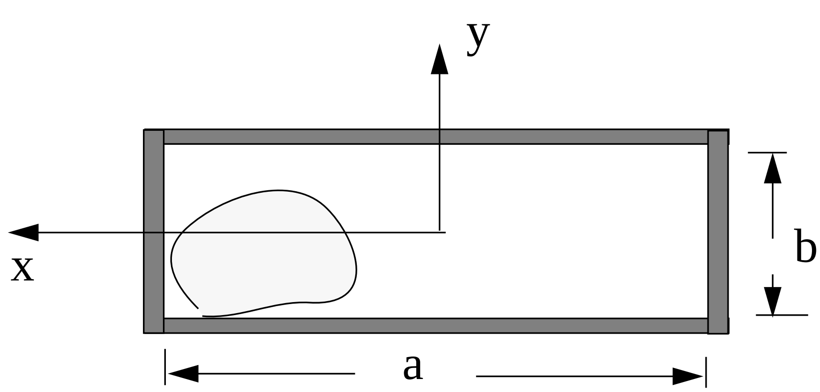

Figure 1(b) Cross-sectional view of rectangular waveguide

Figure 1 Geometry of rectangular waveguide excited by TE_{10} mode

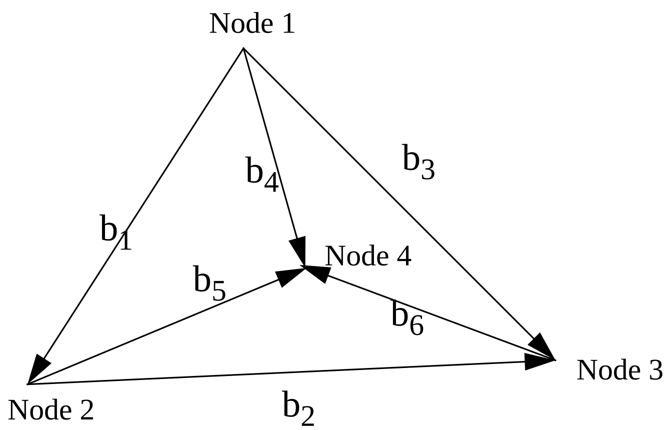

Figure 2 First order tetrahedra element

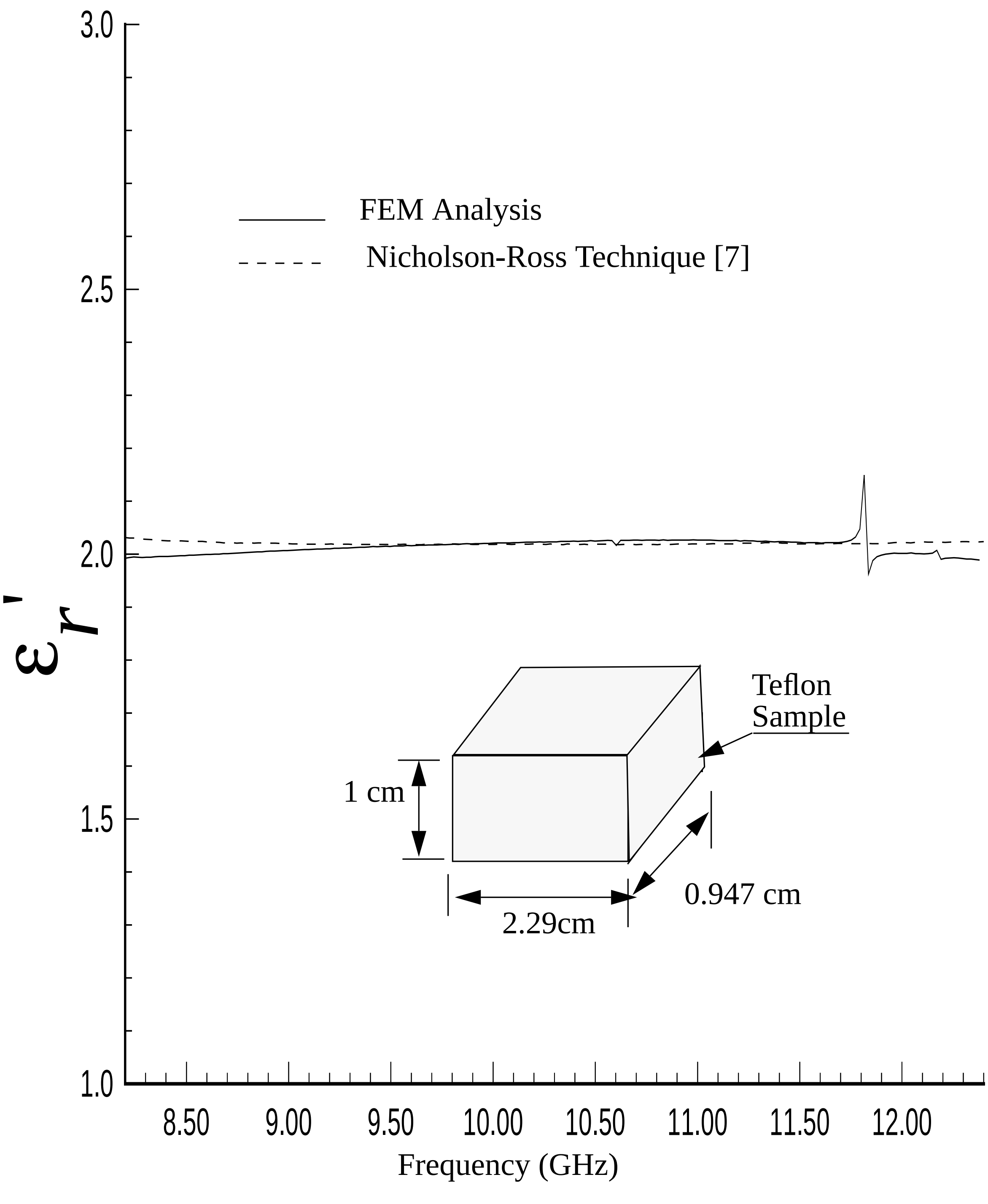

Figure 3(a) Real part of complex permittivity of a Teflon sample over the x-band calculated using two different methods.

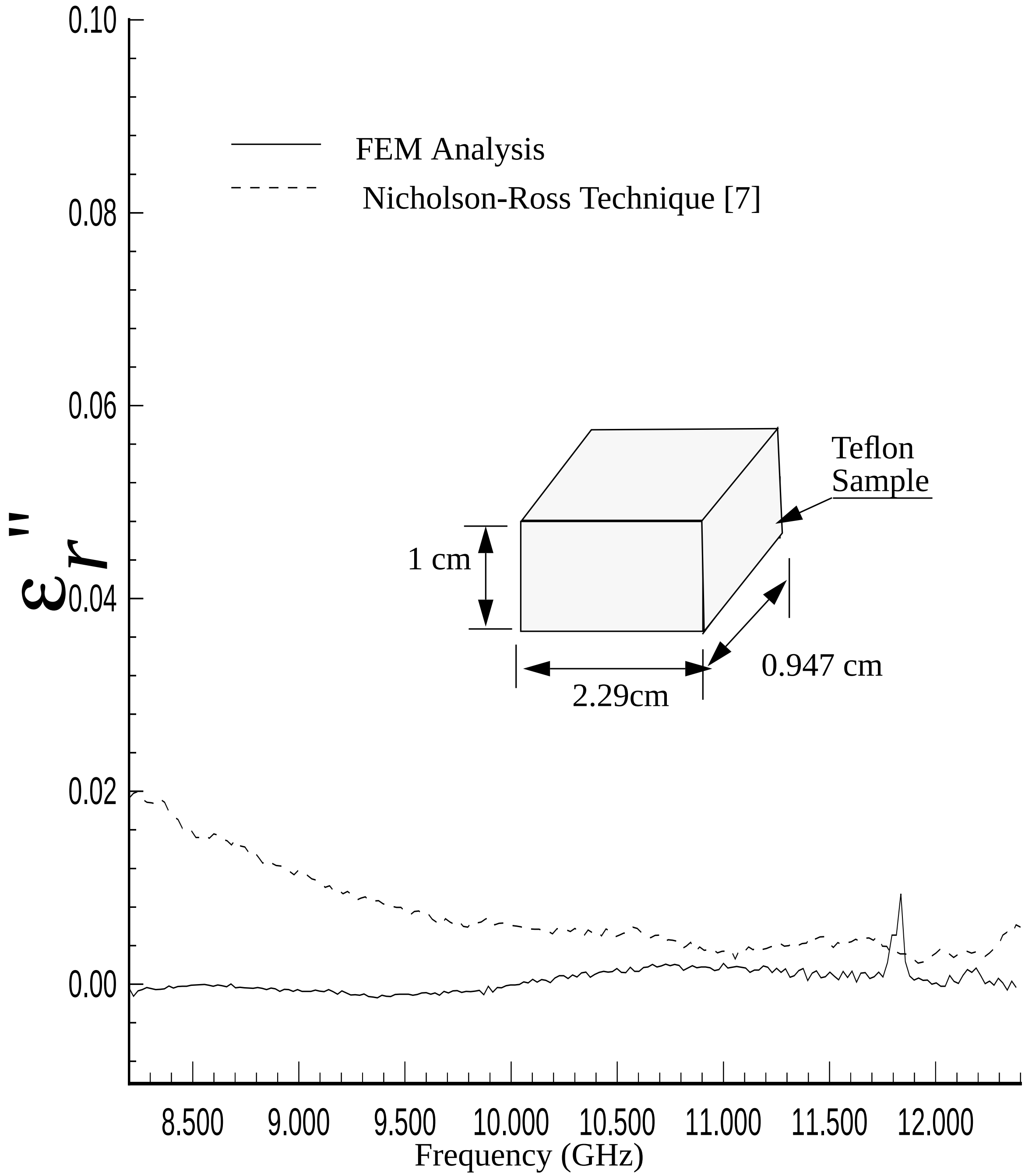

Figure 3(b) Imaginary part of complex permittivity of a Teflon sample over the x-band calculated using two different methods.

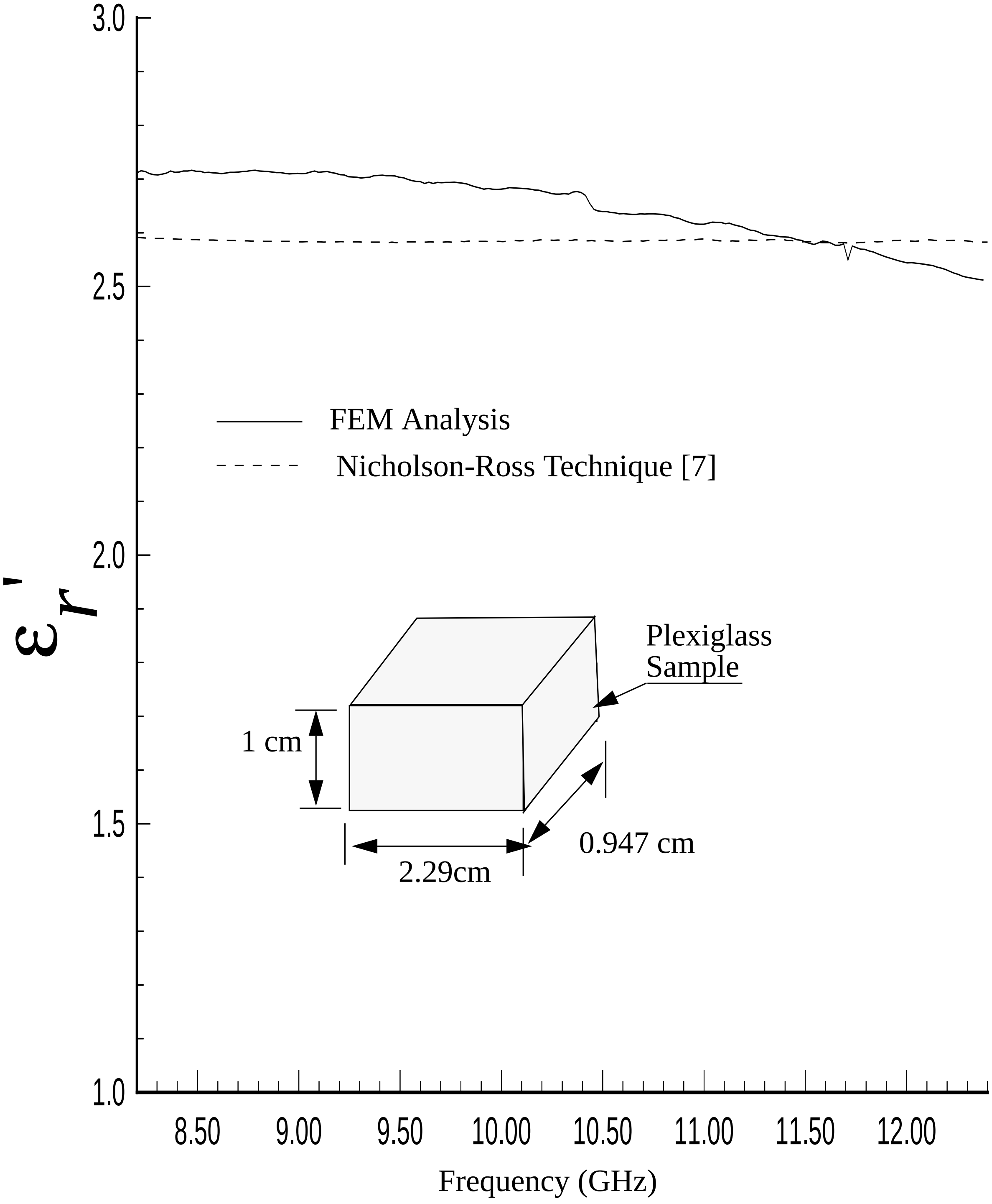

Figure 4(a) Real part of complex permittivity of a plexiglass sample over the x-band calculated using two different methods.

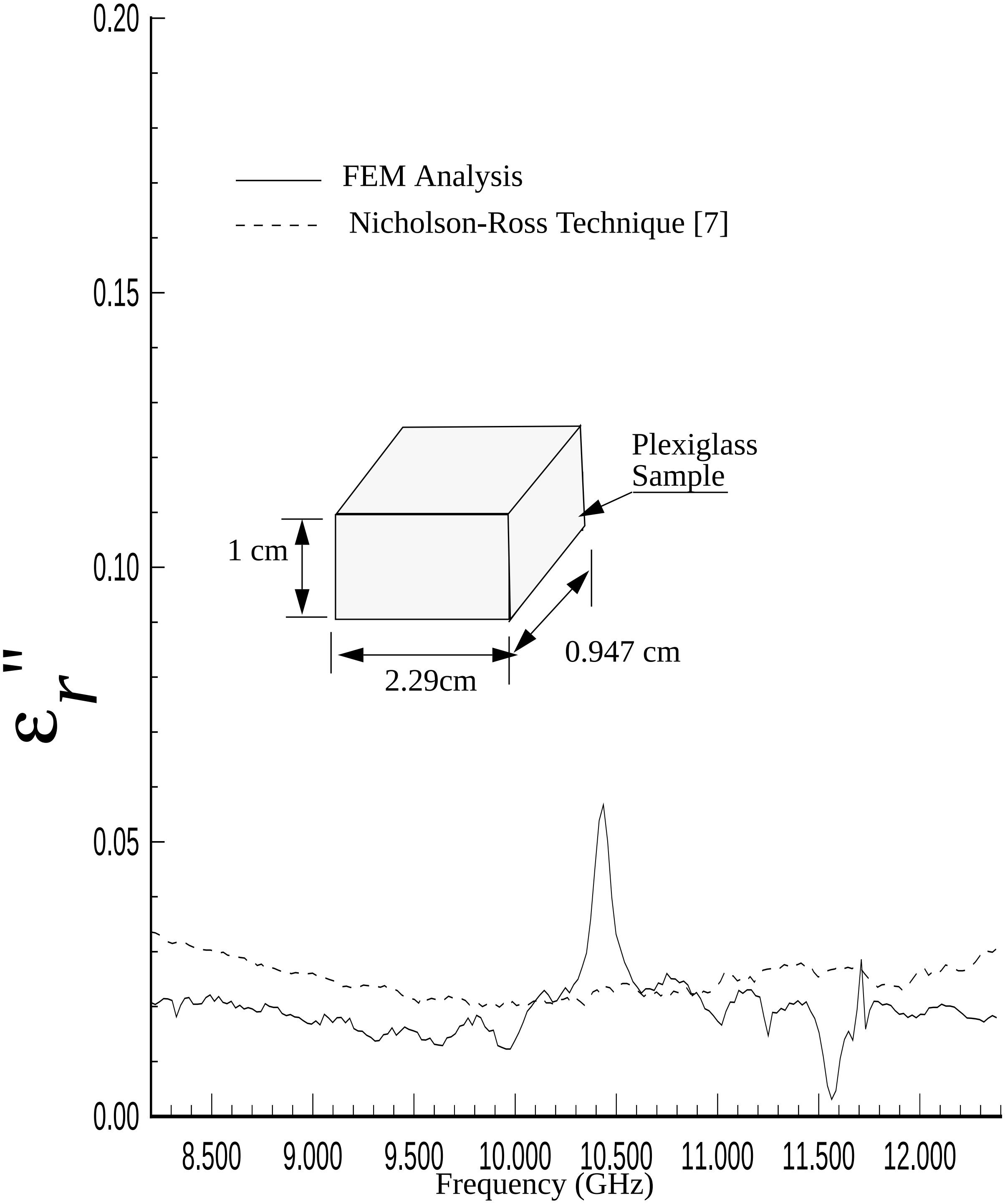

Figure 4(b) Imaginary part of complex permittivity of a plexiglass sample over the x-band calculated using two different methods.

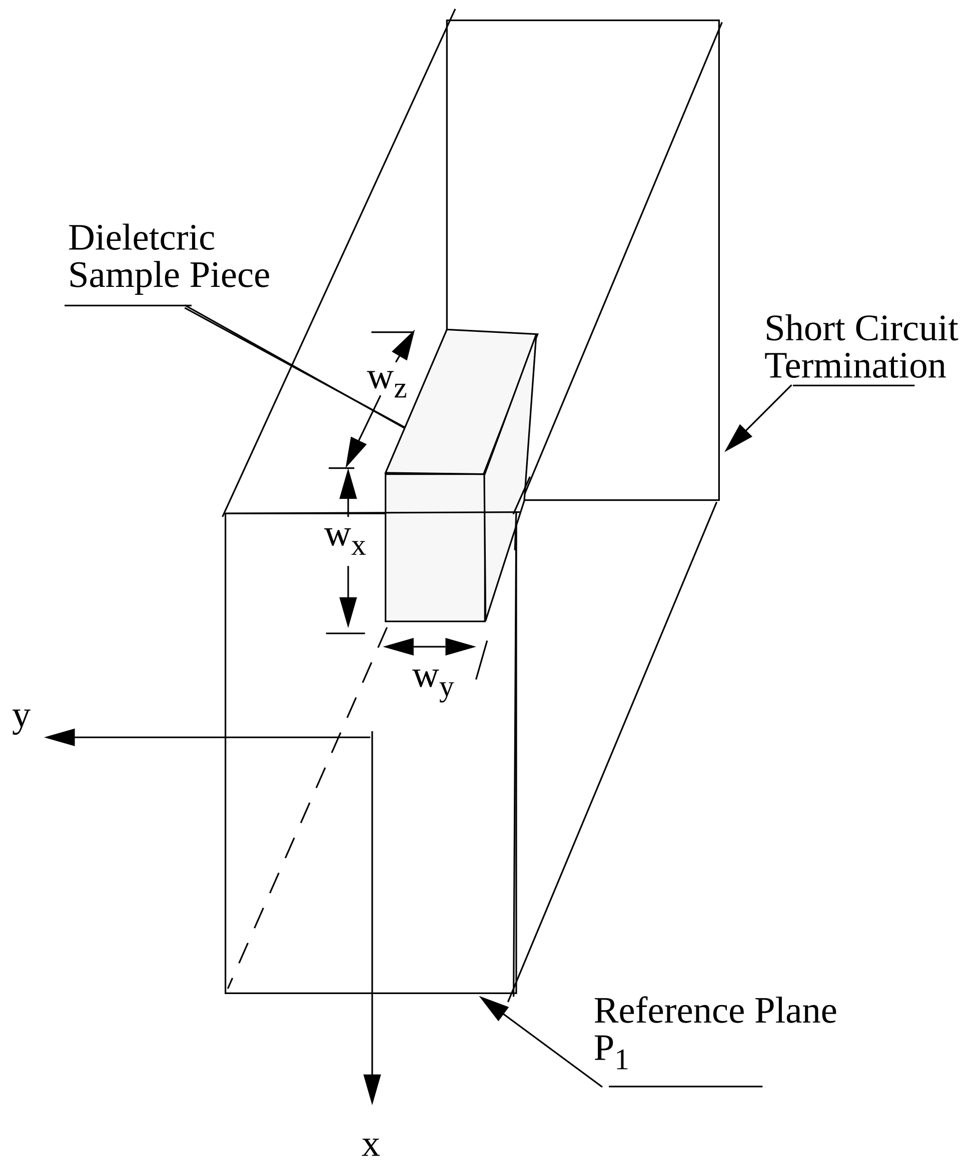

Figure 5 Geometry of the x-band rectangular waveguide holding dielectric sample piece.

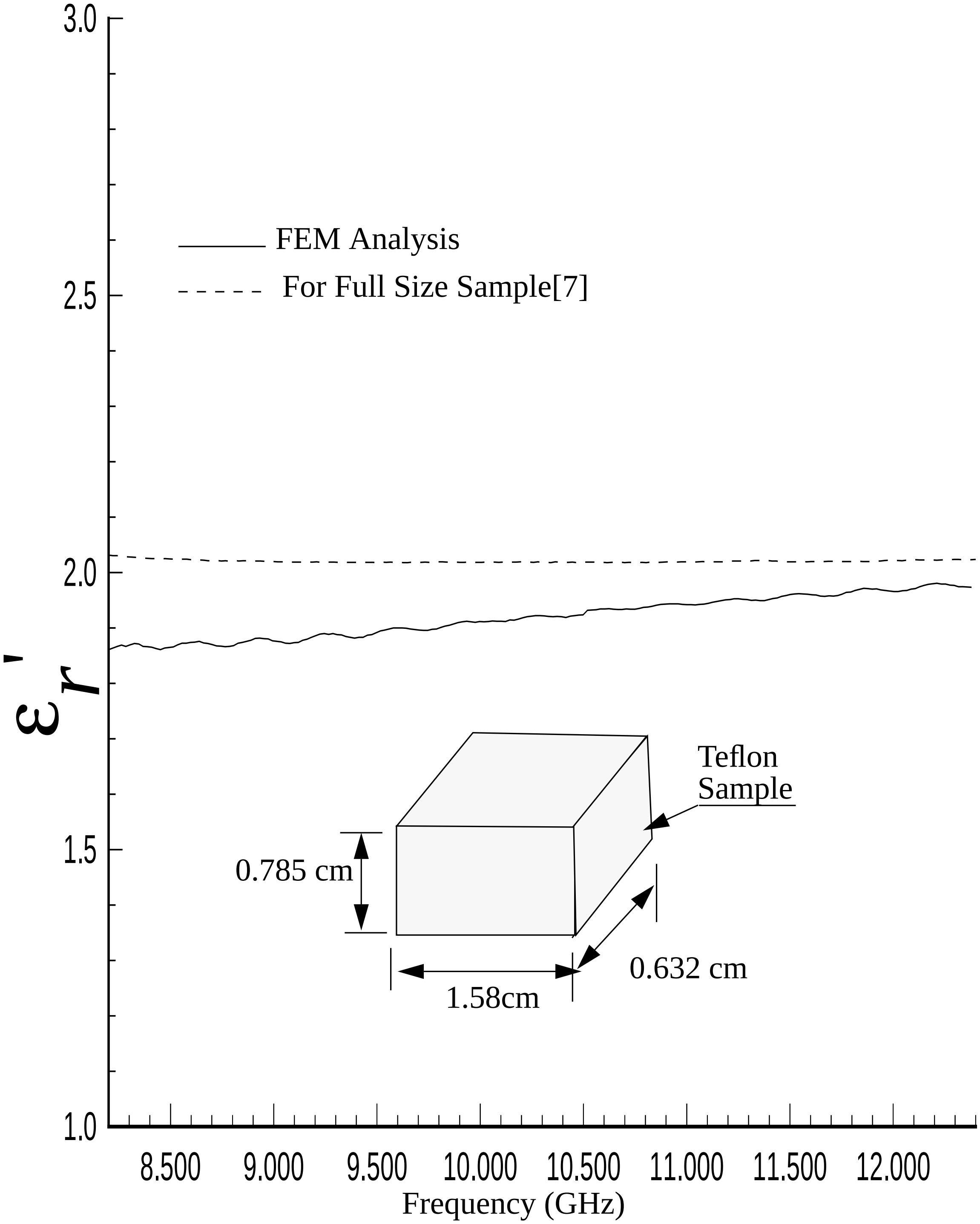

Figure 6 Real part of complex permittivity of a Teflon sample obtained using undersize sample.

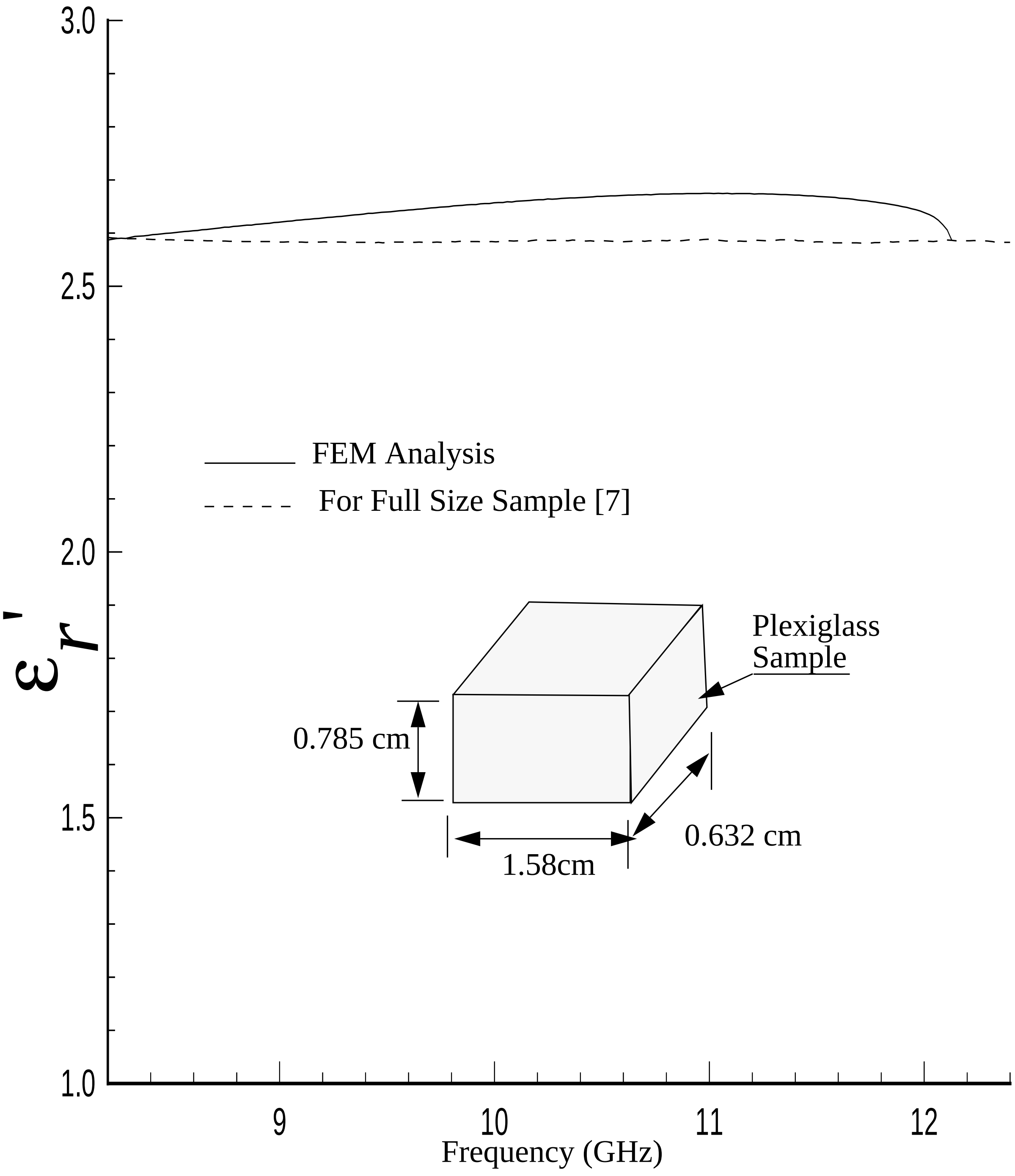

Figure 7 Real part of complex permittivity of a Plexiglass sample obtained using under size sample.